Inference about the plastic behavior of materials from experimental data

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Information about models often comes from many sources. The best use of all the available information comes from an analysis that combines the data from various types of experiments. The Bayesian approach to model analysis provides the necessary tools to combine the various kinds of experimental data with expert judgment about the experiments and models. Such an analysis rests on the quantification of uncertainties in the data and results in estimated uncertainties in the model parameters. The Bayesian approach applies equally well to drawing inferences about a model in situations where a simulation code needs to be used to predict the outcome of a complex physical phenomenon. A desirable feature of Bayesian analysis is that it provides estimates of the uncertainty in predictions made by the simulation code.

I demonstrate the use of Bayesian analysis of a model for characterizing the plastic flow of a metal, the PTW model [1]. The data come from a variety of experiments, from basic measurements of stress versus strain to dynamic experiments involving moderate deformation of metal cylinders, as seen in Taylor impact experiments. In these experiments, a cylinder of the material being tested is propelled into a rigid wall at high velocity, resulting in a significant amount of plastic deformation of the cylinder after it rebounds from the collision. Strain rates in excess of 10^5 per second are routinely attained during the deformation process. The goal of the analysis is to be able to predict the deformation profile in Taylor impact tests involving the specified material, along with estimated uncertainties in the profiles.

The basic data that are used to characterize the plastic behavior of a metal are obtained in quasi-static and Hopkinson bar experiments. In quasi-static tests, a small cylinder of the material is typically squeezed at a constant, relatively slow rate and the change in its height is measured as a function of the load on the cylinder. The measurements are easily converted to stress and strain values. In Hopkinson-bar experiments, a shock wave is transmitted through a thin cylinder of the material and its change in dimensions measured. Although these measurements require the use of a simulation code for interpretation, they are straightforwardly converted to a stress-strain curve at nearly constant strain rate. The strain rates attained in Hopkinson-bar experiments are around 10^3 per second, whereas in quasi-static tests, they are typically less than one per second.

The analysis of these basic data is a fairly straightforward data-fitting problem, albeit a nonlinear one. The approach used here is quite standard. It is based on linearizing the response of the model output with respect to its input. The Jacobian, which characterizes the first-order sensitivities of the model, is used to minimize chi squared, that is, the mean square differences between model predictions and the measured data, normalized to their uncertainties. The Jacobian is also used to estimate the quadratic behavior of chi squared, and hence, the full covariance matrix of the estimated parameters. In the present example, it is necessary to introduce systematic uncertainties to account for sample-to-sample variations in material properties. The treatment of systematic uncertainties in analyzing experimental data is a topic that has not received enough attention in most analyses, let alone in the literature. The present analysis incorporates the systematic uncertainties in a straightforward way.

A major goal of the analysis is to transcribe uncertainties in the data into uncertainties in the fitted parameters. A useful self-consistency check on the results of the analysis involves propagating the uncertainties in the parameters (by means of a Monte Carlo procedure) to uncertainties in the stress-strain curves given by the PTW model. The uncertainty in the curves can be compared to the original data relative to their uncertainties to demonstrate that the analysis makes sense. This test amounts to mapping the uncertainties in the data into the parameters and back again.

Because the physical processes that occur during the impact in a Taylor test involve an integration over various strains and strain rates, its interpretation requires a simulation code. With the aid of a hydrodynamic simulation code, it is possible to use the measurements of the deformed cylinder profile from a Taylor test to make inferences about the PTW model. The Taylor data may be used to adjust the PTW parameters in a way consistent with the analysis of the basic experiments. In Bayesian terminology, the uncertainty distribution from the analysis of the basic experiments may be used as a prior in the analysis of the Taylor data. The result is a posterior distribution that describes the PTW parameter uncertainties that arise from combining the data and their uncertainties from all the experiments. An inherent advantage to this approach is that the data from the Taylor test are consistently combined with those from the basic experiments. Because the strain rates in the Taylor experiment are much higher than those in the Hopkinson bar tests, the inferences made about the PTW plastic flow model cover a larger range of physical conditions. This point is important because if only the data from the basic experiments are used to determine the model parameters, the validity of the model outside their physical operating range (at high strain rates, in this case) would be suspect.

An important question to ask when fitting a model to data is whether the model is consistent with the data. At each stage in the analysis, this question should be answered through a statistical test. When inconsistencies are determined to exist, they need to be addressed. The validity of the model and the data need to be examined. When possible, deficiencies in the model should be corrected by using a more complete model. In the end, the uncertainties assigned to the model parameters must encompass discrepancies with the data. These points will be illustrated with the analysis described above.

References

[1] D. L. Preston, D. L. Tonks, and D. C. Wallace, “Model of plastic deformation for extreme loading conditions,” *J. Appl. Phys.* **93**, pp. 211–220, 2003.